THE MATHEMATICS OF THREAD AND CLOTH CONSTRUCTION: AN HISTORICAL SURVEY.

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As a student in the Department of Textile Industries of the University of Leeds—then the Yorkshire College—I was taught to emulate the successes of my forefathers in the spinning of yarns and in the weaving of fabrics,—and also to perpetuate some of their errors. Empirical work, without any discrimination or "imaginative insight," was the fundamental basis of practically all instruction.

Suggestions had already been thrown out by Beaumont, Murphy, Alçan and others that cloths might be built on scientific lines, but until about 1890 all such suggestions were regarded in this country as the dreams of theorists—and as dreams absolutely futile. Between 1880 and 1890 the late Mr. T. R. Ashenhurst, then of the Bradford Technical College, worked out by direct measurement in some cases and by reasonable inference in other cases, the diameters of woollen, worsted, cotton, silk, etc., yarns, and immediately following the publication of these results and deductions came the rule that

$$\sqrt{\text{yds. per lb.}} = \text{reciprocal of yarn diameter in fractions of an inch}$$

$$(20 \text{ sk. wl.} = \sqrt{20 \times 256} = 60, \text{ or yarn has dia. of } 1/60'');$$

a rule which has since been shown to be the accidental result of taking all the factors of yarn diameter into account—specific gravity of wool, relationship of area of cross-section and length to volume, etc., etc.

Almost immediately following this rule came Ashenhurst's trigonometrical investigations into cloth structures, in which the "angle of curvature" played such an important part.

These two researches together initiated a complete change of attitude on the part of the textile designers to theories of cloth construction; and later years have seen the basic principles, then evolved, practically applied, corrected and re-corrected, until to-day almost all cloth structures are referred to these basic principles.

From 1890 to the present day the tendency has been to accept the possibility of basic rules, which must be qualified by certain factors—empirically ascertained—for kind of fibre, quality of fibre, type of thread structure, type of cloth structure, etc., etc.

It will perhaps clear the ground here to state that in all the confusion of ideas brought forward by later workers in this field, there have really been but two lines of investigation pursued:

First, What were the possibilities of cloth structures geometrically and trigonometrically defined, taking yarns as perfect cylinders, easily bendable but incompressible; and

Second, What variation from these theoretical possibilities would be observable due to (*a*) variations in the thread structure; (*b*) variations in the interlacing or cloth structure following the varying flexibility of the threads employed incident upon the weave floats; and (*c*) variations due to "finish"?

My own method has been to teach rigidly and exactly the principles underlying the interlacing of threads regarded as perfect cylinders (Fig. 1), and then upon this exactitude to impose the variations empirically ascertained.

But even this apparently straightforward method is not perfect. For example, if threads are flexible in one direction, why not in another direction—why should they not bend in any and every direction? (Fig. 2). How can we even theoretically perfectly represent interlacings?

It has taken ten years to make cloth constructors realise that in such a simple structure as $^{2}_{2}$ twill, not only do the threads bend in a direction vertical to the plane of the cloth, but that they also bend in the cloth plane. In

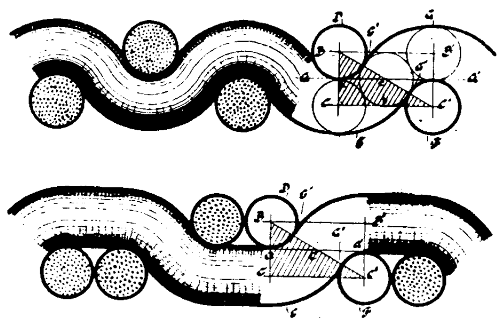

FIG. 1.—From *Textile Design*, Methuen & Co.

passing, it may be remarked that the suggestion given by this recently observed bending is very suggestive to the textile designer—gauze interlacings, for example, being based upon accentuations of the bending here noted (Fig. 2).

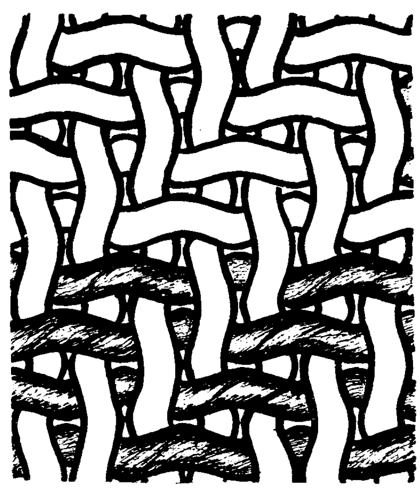

FIG. 2.

It is here interesting to note that from among the workers engaged on the problems referred to, Professor Bradbury of Belfast has principally worked on the theories of interlacing, Mr. Armitage of Huddersfield Technical College has investigated empirically the variations incident upon the natures of woollen and worsted threads, and Mr. Law of Leeds University has experimentally investigated variations in structure incident upon length and relationship of weave float. The final result is that we have now a mathematical basis for the construction of standard cloths, and, better still, that imaginative insight into possible variations which enables us to bring forth new creations.

The success which followed the researches of Mr. Ashenhurst and the work of other researchers into cloth construction naturally suggested that thread construction might also be similarly investigated with advantage. Such investigations have been carried out fairly extensively since 1892.

The development of thread construction theories may be best illustrated by considering relative twist in yarns. Most writers on yarns and cloths realised

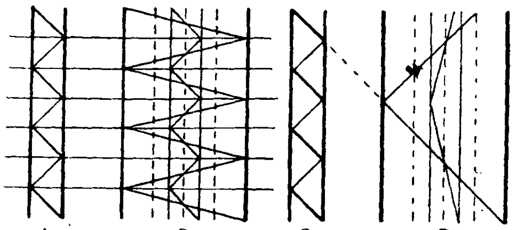

FIG. 3.—From *Cloth Analysis*, Scott, Greenwood & Sons.

that twist might be defined as angular variation from the longitudinal direction of the thread of the fibres composing the thread, and that if this method of indication were adopted it would be necessary to define twist in terms of the diameter, *i.e.* in proportion to the $\sqrt{\text{counts}}$. Thus the "stupid" unimaginative rule that "twist varies inversely as the square of the counts" was put forth. It rested with me personally to show what ought to have been at once apparent, viz., that if the supposed exterior angle of twist is retained the interior structure of counts of different number must be different, and conversely that if the interior structure of the threads were the same the exterior fibre arrangements or twist angles would be different—the two things are incompatible (Fig. 3). It has rested with Mr. Henry Wilkinson of Wyke to show that if any given quality of material (really diameter of fibre) be taken to have a numerical value equal to

$$\frac{1}{q\,1{\cdot}52^{1{\cdot}6}},$$

in which q = quality number, then it is possible to reconcile the apparently irreconcilables, and hence it may be possible within certain limits to make relative changes which are fairly exact.

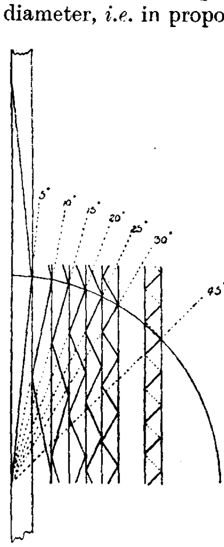

FIG. 4.

The emphasis continually placed by cloth constructors upon "angle of twist" in the yarns employed, led me to the conclusion about ten years ago that twist in yarns should preferably be determined by angle of twist with the longitudinal direction of the thread (Fig. 4). Spinners, however, objected to this definition, contending that turns per 1″ was much the simpler method of definition. To overcome this objection I had a graph on ordinary squared paper prepared, from which the turns per 1″ might be read off for any count of yarn and for any angle (Fig. 5). The curves, it will be observed, present considerable variation from one another, and the graph is much more difficult to draw and read than the simple straight-line graph; but on the suggestion of a member of the engineering staff (Mr. Stelfix) I had these curves plotted out on logarithmic paper (Fig. 6), when straight lines, much more easily to be read, resulted. But better still, Dr. Brodetsky has worked these factors into a "Nomogram" (Fig. 7), which, drawn on a larger scale, seems completely to overcome any practical difficulties in defining twist by "angle" rather than by "turns per inch."

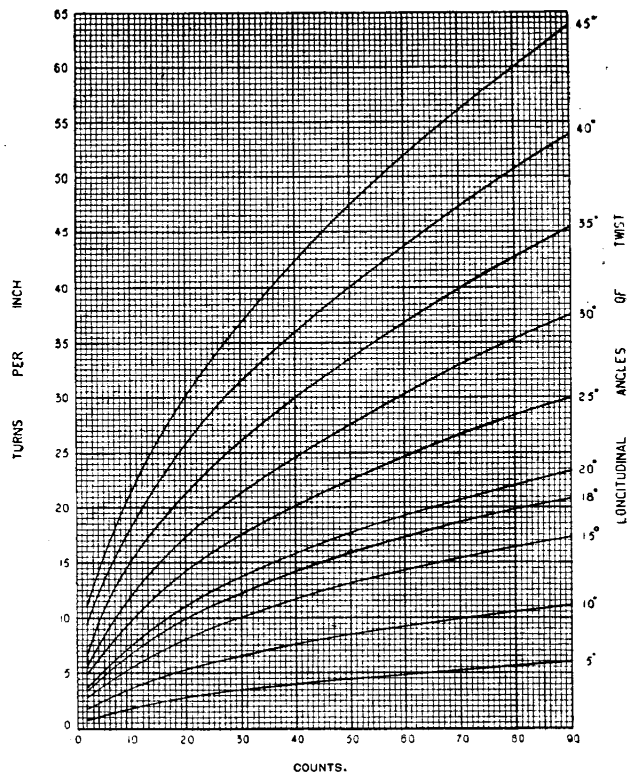
TURNS PER INCH
LONGITUDINAL ANGLES OF TWIST
COUNTS.
45°
40°
35°
30°
25°
20°
18°
15°
10°
5°

FIG. 5.

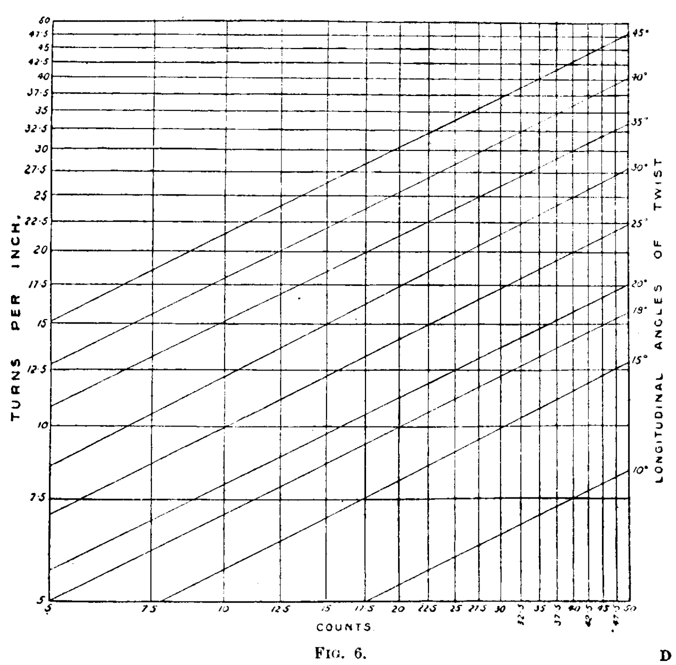
TURNS PER INCH.
LONGITUDINAL ANGLES OF TWIST
COUNTS
45°
40°
35°
30°
25°
20°
19°
15°
10°

FIG. 6.

Following up the idea of "angle of twist," it was soon found necessary to differentiate between "fibre angle" in the case of single yarns, and "twist

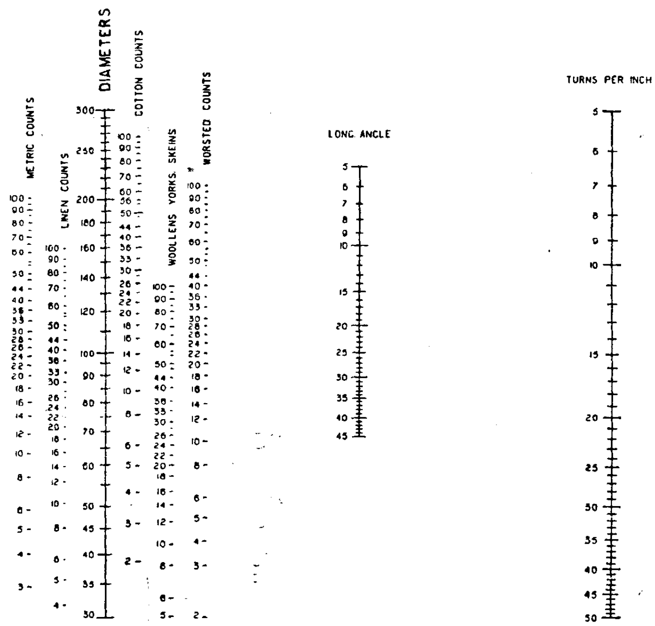

FIG. 7.

angle" in the case of folded yarns. The six types of two-fold twist yarns, as defined by the terms Converse, Straight fibre, and Concurrent twists, thus came to be worked out (Fig. 8). The turns per 1″ for any count of yarn for

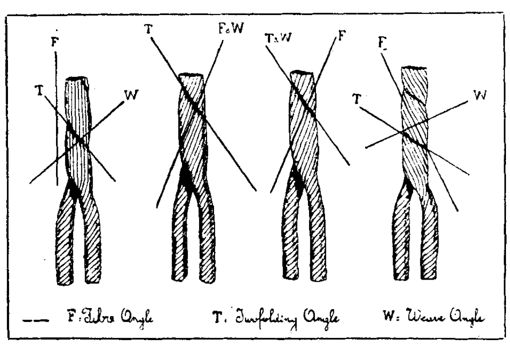

FIG. 8.—From the *Journal of the Leeds Unity Textile Society.*

any required condition have thus been made a simple matter, which can be read off from such a Nomograph (Fig. 7) as that provided. The necessity for the yarn constructor employing imaginative insight is well illustrated in Fig 9. Members of the Mathematical Association will also be interested in other "Nomograms" designed by Dr. Brodetsky to solve particular textile problems.

But the greatest difficulty with reference to scientific work in cloth and thread structure must now be considered. This arises from the many units of measurement employed in the textile industries. Leeds, Bradford, Huddersfield, Manchester, Glasgow—all these textile centres each speak a special textile language, and each has peculiar units of number, length, weight, etc.

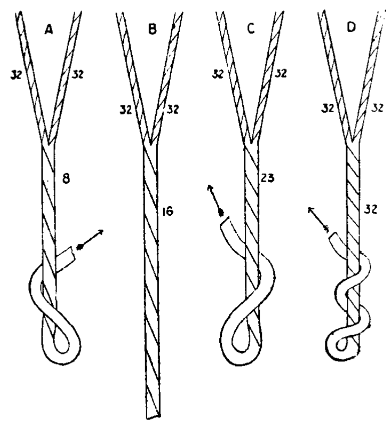

FIG. 9.—From the *Journal of the Leeds Unity Textile Society.*

The numbers in the diagram represent "turns per inch" in the single and twofold yarns. With the first conditions (left) the single twist dominates and causes the twofold yarn to twist upon itself as shown. With the second conditions single twist and twofold twist balance one another, and with the third conditions the twofold twist dominates.

With the addition of experimental laboratories to our textile factories the metric system must also be included, and it is to be hoped that ultimately the metric system will be adopted not only in the laboratory but throughout the industry.

Perhaps, in conclusion, one example illustrating the numerical difficulties involved in mounting a loom to weave a figured fabric may be given:

(*a*) The count of yarn (20s, 30s, 40s, etc.) may be stated on a group unit basis of 256 or 560 or 840, etc., etc.

(*b*) The threads may be stated in group-units of 38 or 40 or 42 threads—a relic of counting by the score.

(*c*) The weight of the warp and weft may be stated in warterns (6 lbs.), *i.e.* quarterns of the old 24 lb. stone.

(*d*) The length of the warp in the Leeds district will be given in "strings" of 10 feet.

(*e*) The heald shafts will group the threads in 4s, or 6s, or 8s for Tappet looms, in any grouping up to 36 for Dobby looms, or up to 1200 for Jacquard looms.

(*f*) The colour pattern of the warp may be grouped in anything from 2 to say 200.

(*g*) The reed will group its threads in 2, 3, 4, 5, 6, 7s, or 8s; and more important still, the (*e*), (*f*) and (*g*) groupings must be exactly adjusted the one to the other. Anyone having these varying groupings to adjust—the one to the other—will realise what an iniquitous imposition the multiplication table as ordinarily taught in our schools is—how it enervates and hides rather than stimulates and reveals.

Thus it will be evident that thread and cloth construction are no "clerking job," but specially require that imaginative insight which surely leads to intelligent manipulation and control of all the factors involved in the solution of the problems presented.

A. F. B.